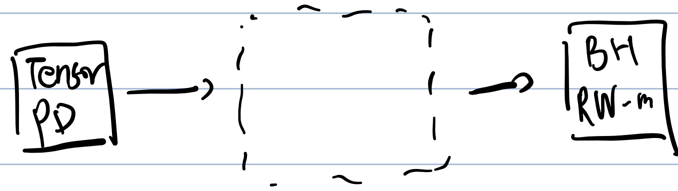
space time curvature.



equivalence principle $\xrightarrow{GR}$ local coordinate sys

gravity → force / field.

express all spacetime in terms of local coordinate.

gravity does not exist

↓

should be a manifestation of spacetime curvature.

$$\underline{ds^2} \approx \underset{\text{(Minkowski)}}{\underline{\eta_{\mu\nu}}}\, \underline{dx^\mu}\, \underline{dx^\nu}$$

$$\frac{Du^\mu}{D\tau} = 0. \quad \text{eom}$$

Local inertia coordinate.

close to any point P. we can always find / construct

coordinates X^μ on pseudo-Euclidean 4D manifold

s.t.

$$g_{\mu\nu}(p) = \eta_{\mu\nu} \qquad (\partial_\rho g_{\mu\nu})_p = 0.$$

Physically, these coordinates correspond to a free-falling, non-rotating, Cartesian ref frame.

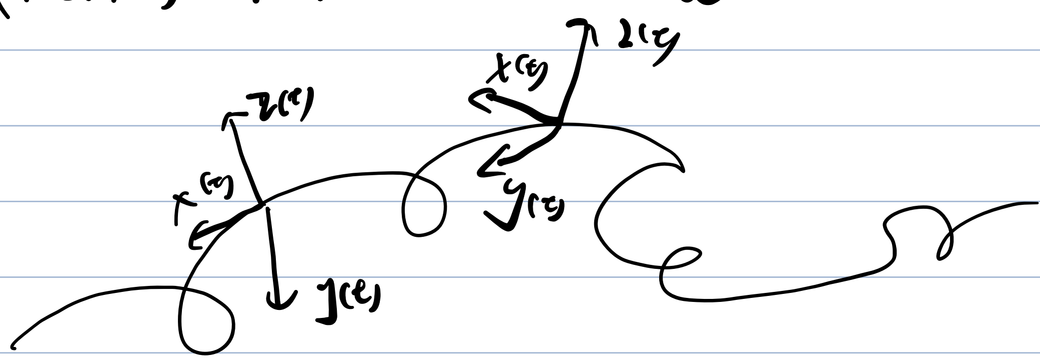
=> only valid in some limited region.

$$e_\mu \equiv \frac{\partial}{\partial x^\mu} \qquad g(e_\mu, e_\nu) = \eta_{\mu\nu}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2}\left(\frac{\partial^2 g_{\mu\nu}}{\partial x^\rho \partial x^\sigma}\right)_p [x^\rho - x^\rho(p)][x^\sigma - x^\sigma(p)] + \dots$$

$\rho \quad \sigma$

(Fermi)-normal coordinate.



Newton limit for free-falling particles

→ Newtonian sys <~> weakly curved st.

$$g_{\mu\nu} = \boxed{\eta_{\mu\nu}} + \boxed{h_{\mu\nu}} \qquad |h_{\mu\nu}| < 1$$

assume $\frac{\partial h_{\mu\nu}}{\partial x^0} = 0.$ → static gravitational field.

η_{10} $\qquad g_{10} = \cancel{\eta_{10}} + h_{10}$

slow-moving particles ⇒

$$\left|\frac{dx^i}{dt}\right| \ll c. \rightarrow \frac{dx^0}{dt}.$$

$$\Downarrow$$

$$\left|\frac{dx^i}{d\tau}\right| \ll \frac{dx^0}{d\tau}$$

Geodesic eqn. $\qquad \frac{x^i}{d\tau}$

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\rho\sigma}\frac{dx^\rho}{d\tau}\frac{dx^\sigma}{d\tau} = 0.$$

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{00}c^2\left(\frac{dt}{d\tau}\right)^2 \simeq 0.$$

$$\Gamma^\mu_{00} = \frac{1}{2}g^{\mu\nu}\left(2\frac{\partial g_{\nu 0}}{\partial x^0} - \frac{\partial g_{00}}{\partial x^\nu}\right)$$

$$\approx -\frac{1}{2}\sum_i \eta^{\mu i}\frac{\partial h_{00}}{\partial x^i}$$

$$\Gamma^0_{00} \approx 0. \qquad \Gamma^i_{00} \approx \frac{1}{2}\frac{\partial h_{00}}{\partial x^i}$$

$$\frac{d^2 x^i}{d\tau^2} \approx -\frac{c^2}{2}\frac{\partial h_{00}}{\partial x^i}\left(\frac{dt}{d\tau}\right)^2$$

$$a = \frac{d^2 x^i}{dt^2} \approx -\frac{c^2}{2}\frac{\partial h_{00}}{\partial x^i}$$

Newtonian eom

$$\frac{d^2 x^i}{dt^2} = \frac{-\partial \Phi}{\partial x^i}$$

pot

$$h_{00} \approx \frac{2\Phi}{c^2}$$

$$g_{00} \approx (1 + \frac{2\Phi}{c^2})$$

$$\left|\frac{2\Phi}{c^2}\right| \ll 1$$

$$\frac{\Phi}{} \ll 1$$

c^2

Intrinsic curvature of a manifold.

$$ds^2 = \varepsilon_1 (dx^1)^2 + \varepsilon_2 (dx^2)^2 + \cdots \varepsilon_N (dx^N)^2$$

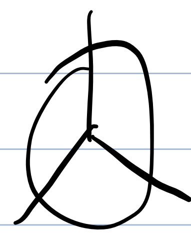
$$\varepsilon_i = \pm 1$$

spacetime. → is it flat?

$$\boxed{ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2}$$

↓

express in Cartesian coords $ds^2 = (dx - a_1)^2 + \cdots$



Riemann curvature tensor.

(a measure of curvature)

Covariant derivative.

$$\nabla_a \nabla_b \phi = \nabla_b \nabla_a \phi.$$

scalar fn.

Consider a dual-vector field V_a.

$$\nabla_a \nabla_b V_c = \partial_a(\nabla_b V_c) - \Gamma^d_{ab} \nabla_d V_c - \Gamma^d_{ac} \nabla_b V_d$$

$$= \partial_a(\partial_b V_c - \Gamma^d_{bc} V_d) - \Gamma^d_{ab}(\partial_d V_c - \Gamma^e_{dc} V_e) - \Gamma^d_{ac}(\partial_b V_d - \Gamma^e_{bd} V_e)$$

$$\nabla_a \nabla_b V_c - \nabla_b \nabla_a V_c = -\partial_a \Gamma^d_{bc} V_d + \partial_b \Gamma^d_{ac} V_d + \Gamma^e_{ac} \Gamma^d_{be} V_d - \Gamma^e_{bc} \Gamma^d_{ae} V_d$$

$$= R_{abc}{}^d V_d$$

$$R_{abc}{}^d = -\partial_a \Gamma^d_{bc} + \partial_b \Gamma^d_{ac} + \Gamma^e_{ac} \Gamma^d_{be} - \Gamma^e_{bc} \Gamma^d_{ae}$$

type (1, 3) tensor.

Riemann curvature tensor

metric

$\partial \Gamma$

flat $\Longleftrightarrow$ Riemann $= 0$.