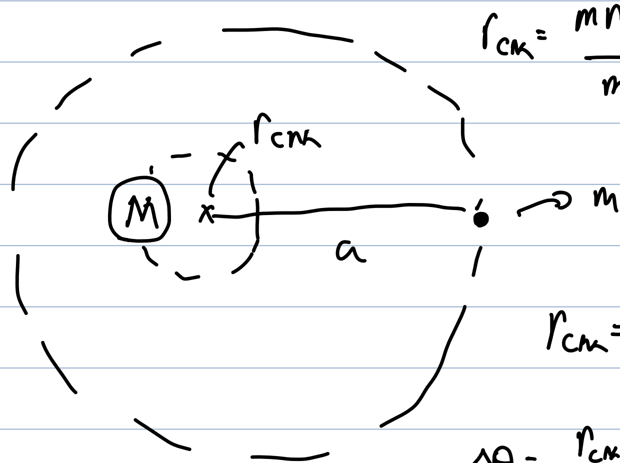
$$\Delta\theta = \frac{a}{d}\,\frac{m}{M}$$



$$r_{cm} = \frac{mr + MR}{m+M}$$

$(R=0)$

$$r_{cm} = \frac{mr}{m+M}$$

$$\Delta\theta = \frac{r_{cm}}{d} = \frac{m}{m+M}\,\frac{r}{d}$$

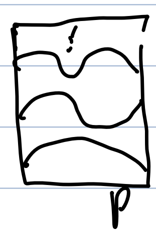
$$= \frac{m}{m+M}\,\frac{a}{d}$$

$M \gg m$

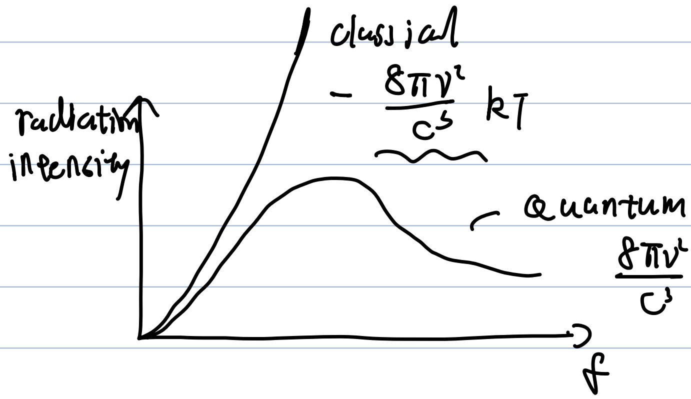
$$= \frac{m}{M}\,\frac{a}{d}$$

Ultraviolet Catastrophe.

Black radiation refer to an object or system absorbs all radiation incident upon it and re-radiate it's independent upon the type of radiation which is incident u on it.



	mode per unit freq per unit vol	average E per mode.
classical	$\frac{8\pi\nu^2}{c^3}$	kT
Quantum	$\frac{8\pi\nu^2}{c^3}$	$\frac{h\nu}{e^{\frac{h\nu}{kT}}-1}$

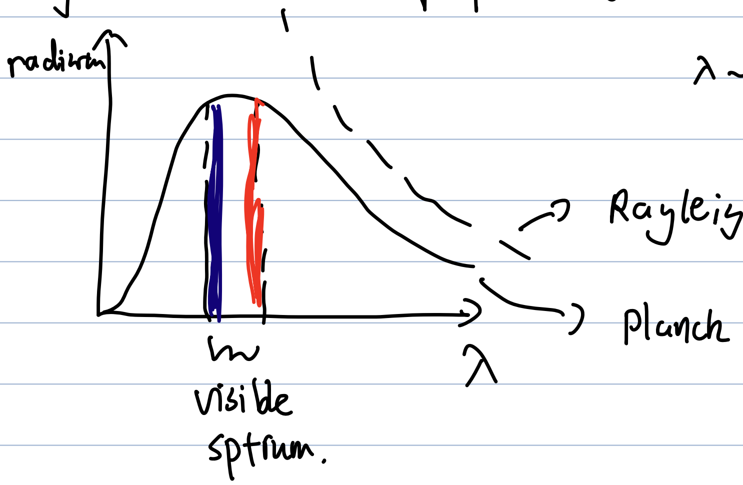


$-\ \frac{8\pi\nu^2}{c^3} kT$

$-\ \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{\frac{h\nu}{kT}}-1}$

probability of occupying certain mode.

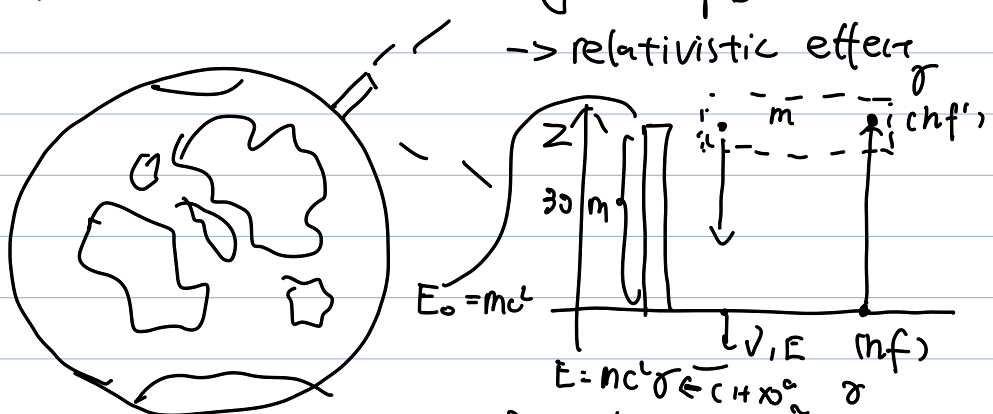
The amount of radiation emitted in a given freq range should be proportional to the # of modes.



$\lambda \sim \frac{1}{f}$.

Einstein's tower (thought expt)

-> relativistic effect



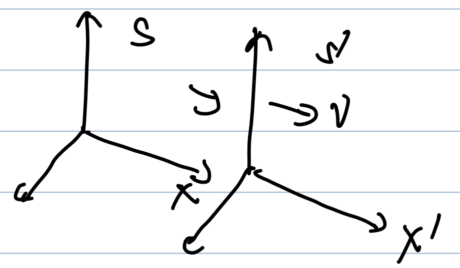
$E = mc^2\gamma \leftarrow (1+x)^a \quad \gamma$

$\Delta f = f' - f\ ? \quad \gamma \approx 1+ax$

initial $f = 4.3\times10^{14}$ Hz

Galilean transformation

$S \quad S' \quad v$



invariant Δt.

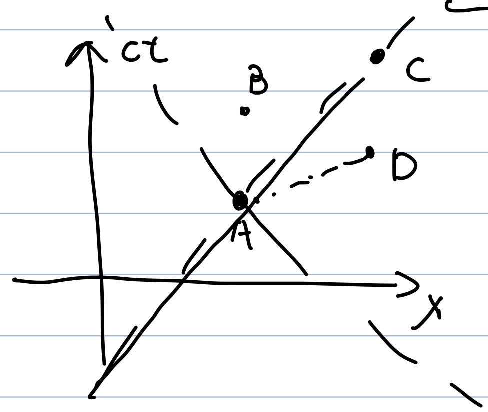
Lorentz transformation.

$$ct' = \gamma(ct - \beta x) \qquad x' = \gamma(x - \beta ct)$$

$$\beta = \frac{v}{c} \qquad \gamma \equiv (1-\beta^2)^{-\frac{1}{2}} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

invariant

$$\Delta s^2 = c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2.$$ (Minkowski space time)



AB -> $\Delta s^2 > 0$. timelike interval

AC => null (light like) interval $\Delta s^2 = 0$.

AD -> space like $\Delta s^2 < 0$.

length contraction

$$l = l_0/\gamma$$

time dilation

$$T = T_0\gamma$$

rapidity φ $\qquad \beta = \tanh\varphi$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \cosh\varphi \qquad \gamma\beta = \sinh\varphi.$$

$$ct' = ct\cosh\varphi - x\sinh\varphi.$$
$$x' = -ct\sinh\varphi + x\cosh\varphi.$$
$$y' = y \qquad z' = z.$$

hyperbolic ✓ trigonometric ✗.

$$\Delta s^2 = (c\Delta t)^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

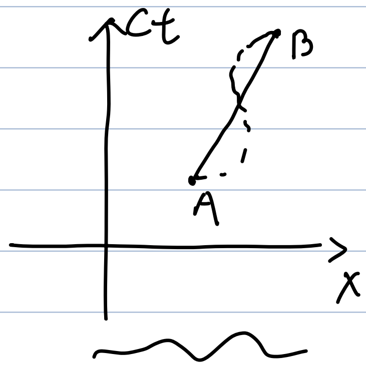
More general

(1) $ct \neq x \neq y \neq z = 0$ $x' \neq y' \neq \cdots = 0$.

(2) vel → x vel → y, z

(3) spatial axis S, S′ may not align

=> inhomogeneous Lorentz transf / Poincaré transf.



for general path

we must express the intrinsic geometry of Minkowski spacetime in infinitesimal form

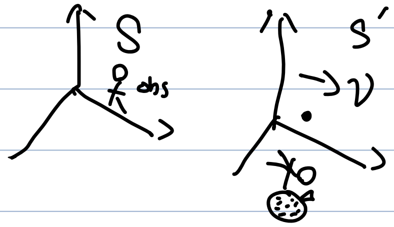
$\Delta s \to ds$

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

$$\Delta S = \int_A^B ds.$$

Proper time.

measured by an ideal clock carried by observer comoving with particle.



increment in proper time $d\tau$ is increment in time in instantaneous rest frame of the particle

$$dx' = dy' = dz' = 0.$$

$$ds^2 = c^2 d\tau^2 - dx'^2 - dy'^2 - dz'^2 = c^2 d\tau^2$$

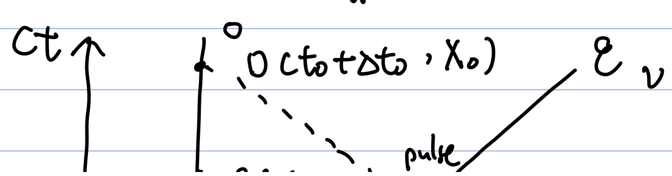
$$= c^2 dt^2 - dx^2 - dy^2 - dz^2$$

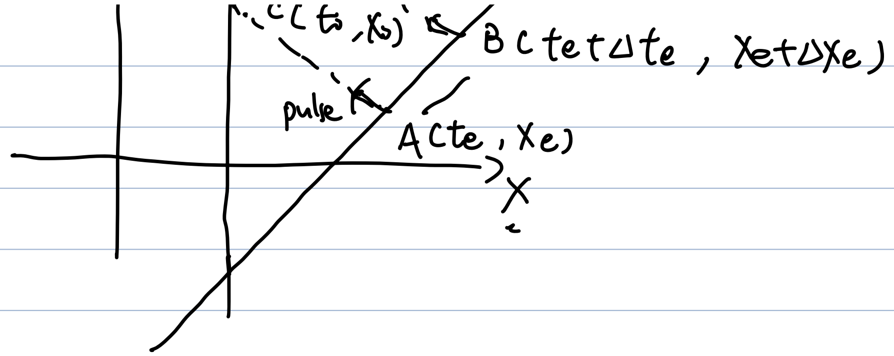
$$d\tau^2 = dt^2 - \frac{1}{c^2} dx^2 - \frac{1}{c^2} dy^2 - \frac{1}{c^2} dz^2$$

$$\boxed{d\tau = \frac{dt}{\gamma_v}}$$

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Relativistic Doppler effect.



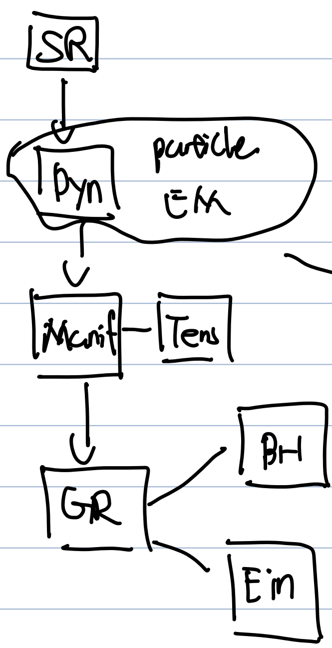


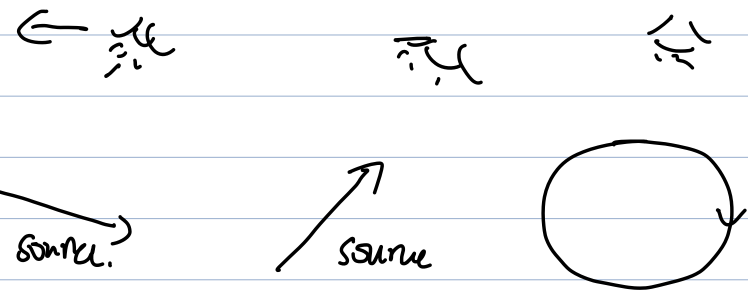
$$\Delta\tau_{AB} = \frac{\Delta t_e}{\gamma_v}$$

$$\Delta t_0 = \frac{v\Delta t_e}{c} + \Delta t_e$$

$$\frac{\Delta\tau_{AB}}{\Delta\tau_{CD}} = \frac{\cancel{\Delta t_e}}{\gamma} \Big/ \left(\frac{v}{c}+1\right)\cancel{\Delta t_e} = \frac{\sqrt{1-\left(\frac{v}{c}\right)^2}}{1+\frac{v}{c}} = \frac{\sqrt{\left(1-\frac{v}{c}\right)\left(1+\frac{v}{c}\right)}}{\sqrt{\left(1+\frac{v}{c}\right)\left(1+\frac{v}{c}\right)}}$$

$$= \sqrt{\frac{1-\beta}{1+\beta}}$$





$$\frac{1}{c}\langle 0|(a+a^\dagger)^4|0\rangle - 3$$

1 2 1

1 3 3 1

1 4 6 4 1

$$\downarrow$$

$a^3 a^{3\dagger} |0\rangle$

$a^3 |3\rangle$

$\langle 0|0\rangle \neq 0.$