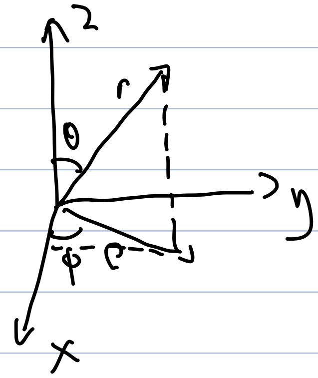


cartesian (x, y, z)

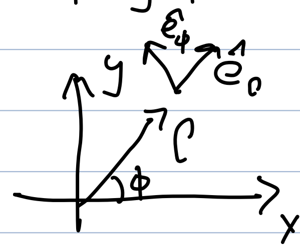
cylindrical (ρ, ϕ, z)

spherical (r, θ, ϕ)

$x =$ $\rho\cos\phi$ =	$r\sin\theta\cos\phi$	
$y =$ $\rho\sin\phi$ =	$r\sin\theta\sin\phi$	
$z =$ z =	$r\cos\theta$	
ca	cy	sp

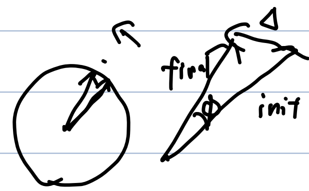
$\vec{F} = m\vec{a}$

$a = \frac{d^2\vec{r}}{dt^2} = \ddot{r}$



$r = \rho\,\hat{e}_\rho$

↳ unit vector



$\dot{r} = \dot{\rho}\,\hat{e}_\rho + \rho\,\dot{\hat{e}}_\rho$

$d\hat{e}_\rho = d\phi\,\hat{e}_\phi$

$\frac{d\hat{e}_\rho}{dt} = \frac{d\phi}{dt}\,\hat{e}_\phi$

$$\boxed{\dot{\hat{e}}_\rho = \underbrace{\dot{\phi}}_{(\omega)}\,\hat{e}_\phi}$$

$$\boxed{\dot{\hat{e}}_\phi = -\dot{\phi}\,\hat{e}_\rho}$$

⇒ $\dot{x} = \frac{dx}{dt}$

$\ddot{x} = \frac{d^2x}{dt^2}$

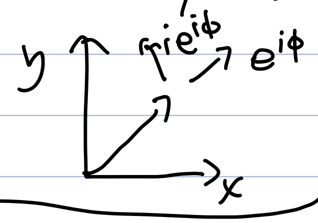
$$\ddot{r} = \ddot{\rho}\hat{e}_\rho + \dot{\rho}\dot{\hat{e}}_\rho + \dot{\rho}\dot{\phi}\hat{e}_\phi + \rho\ddot{\phi}\hat{e}_\phi + \rho\dot{\phi}\dot{\hat{e}}_\phi$$

$$= \underbrace{(\ddot{\rho} - \rho\dot{\phi}^2)}_{\text{radial}}\hat{e}_\rho + \underbrace{(2\dot{\rho}\dot{\phi} + \rho\ddot{\phi})}_{\text{transverse}}\hat{e}_\phi$$

$$\overbrace{\frac{1}{\rho}\frac{d}{dt}(\underbrace{\rho^2\dot{\phi}}_{\text{angular mom per unit mass}})}^{} = 0.$$

complex plane

$z = x + iy = \rho e^{i\phi}$



$$\int \frac{d}{dt}(\rho^2\dot{\phi}) = \int 0$$

$$|\vec{L}| = \rho^2\dot{\phi} = \text{const}$$

$$\vec{L} = \underset{\text{position}}{\vec{r}} \times \underset{\underbrace{\text{momentum}}_{\text{mom}}}{\vec{p}}$$

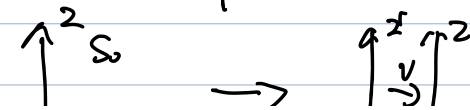
Suppose S_0 frame. $m\ddot{r}_0 = F$

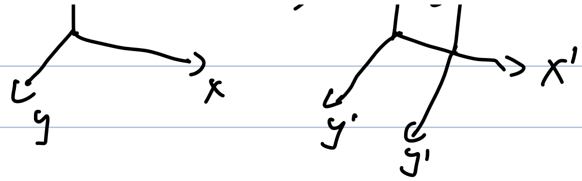
apparent eqn of motion in a moving frame?

case 1 $\quad r = r_0 - R(t) \quad$ axes parallel (S_0 & S)
init $t = t_0$

$$\ddot{r} = \ddot{r}_0 - \underbrace{\ddot{R}(t)}_{0}$$

$$m\ddot{r} = \ddot{r}_0 m = F \quad \Rightarrow \text{Galilean transformation}$$





In general $\ddot{R}(t) \neq 0$: $\quad m\ddot{r} = m\ddot{r}_0 - m\ddot{R}$

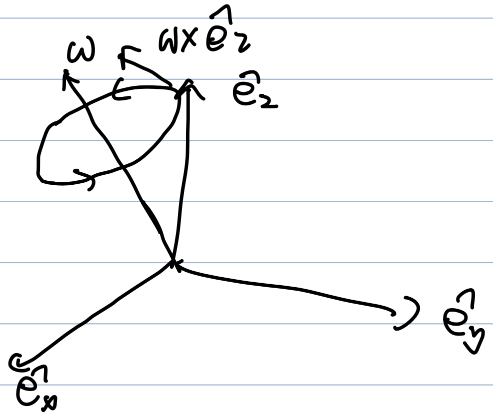
$$= F - \underbrace{m\ddot{R}}_{\sim \text{fictitious force.}}$$

(a) associated with accelerrated frame.

(b) proportional to mass

Note: According to $\underbrace{\text{GR}}_{\text{general relativity}}$, gravity is a fictitious force.

Case 2 Rotating frame.



Frame S rotates with angular vel ω

$$\dot{\hat{e}}_z = \omega \times \hat{e}_z$$

$t_0, t = 0$.

$$\underline{r_0} = \underbrace{x\hat{e}_x}_{} + y\hat{e}_y + z\hat{e}_z = r.$$

$$\dot{r}_0 = \dot{x}\hat{e}_x + x\dot{\hat{e}}_x + \dot{y}\hat{e}_y + y\dot{\hat{e}}_y + \dot{z}\hat{e}_z + z\dot{\hat{e}}_z$$

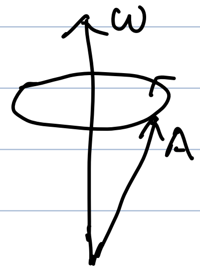
$$= \underbrace{v}_{\text{apparent vel in } S} + \omega \times r$$

$$= \underbrace{v + \omega \times r}.$$

$x = x(t)$ $\qquad \ddot{r}_0 = a + 2\omega \times v + \omega \times (\omega \times r)$

$$ma = \underbrace{m\ddot{r}_0}_{F} \underbrace{- 2m(\omega \times v) - m\,\omega \times (\omega \times r)}_{\text{fictitious force.}}$$

$\omega \otimes x$



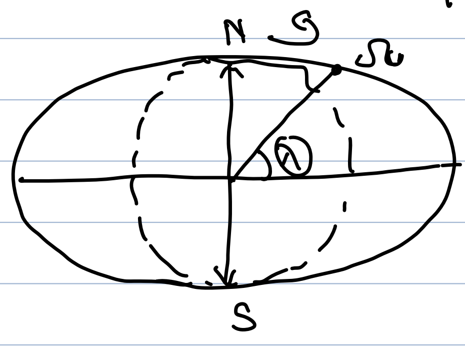
$$\left[\frac{dA}{dt}\right]_{S_0} = \left[\frac{dA}{dt}\right]_S + \omega \times A.$$

$$\left[\frac{d^2 r_0}{dt^2}\right]_{S_0} = \left(\left[\frac{d}{dt}\right]_S + \omega \times\right)\left(\left[\frac{dr}{dt}\right]_S + \omega \times r\right)$$

$\frac{dy}{dx}$

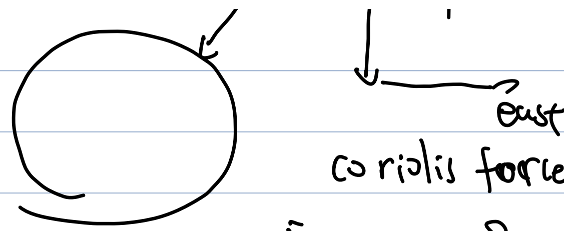
$\omega(t)$ $\quad \omega$ is a func of time.

$$ma = F \underbrace{- 2m(\omega \times v)}_{\text{Coriolis force}} \underbrace{- m\,\omega \times (\omega \times r)}_{\text{centrifugal force.}} \underbrace{- m\dot{\omega} \times r}_{\text{Euler force}}$$



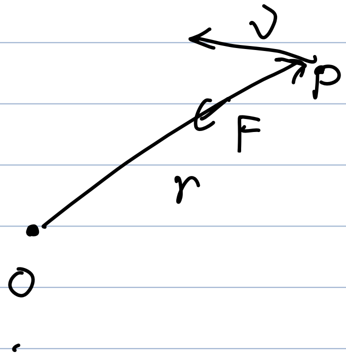
$$\frac{\Omega^2 R}{g} \sim \frac{1}{300}.$$

$| \to F$



coriolis force

$F = 2mv\Omega\cos\lambda$



$m r^2 \dot{\phi} = J = \text{constant}$

$E = U + \frac{1}{2} m \dot{r}^2 + \frac{1}{2} m r^2 \dot{\phi}^2$

$= \underbrace{U + \frac{J^2}{2mr^2}}_{U_{eff}} + \frac{1}{2} m \dot{r}^2$

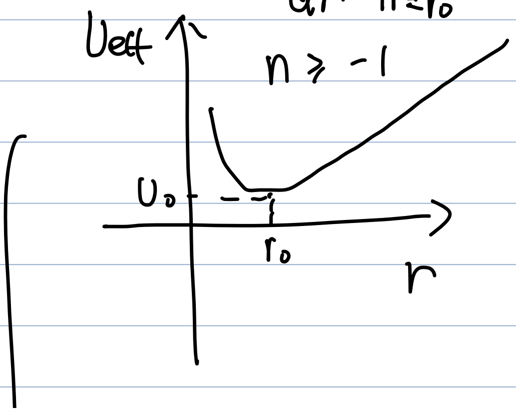
Let's consider orbital force $\boxed{F = -Ar^n}$

$A > 0.$

$(-\nabla U = F)$

$U_{eff} = \frac{A r^{n+1}}{n+1} + \frac{J^2}{2mr^2}$ (! $n = -1$ $\log r$!)

$\left.\frac{dU_{eff}}{dr}\right|_{r=r_0} = 0.$



$r \to \infty$ $U \uparrow$

stable.

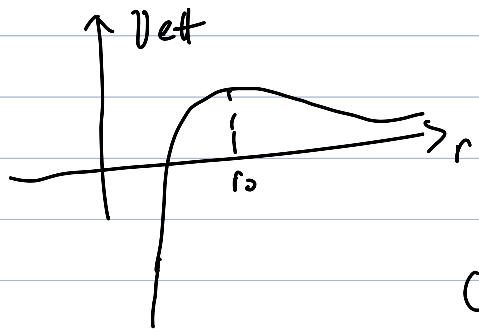
bounded.

$-3 < n < -1$

$r \to \infty \quad U \to 0$

bound ?
unbound

$n < -3$



$r \to 0.$

attraction overcomes
centrifugal repulsion
not stable

(BH)

$$E = -\frac{\overbrace{GmM}^{\to A}}{r} + \frac{J^2}{2mr^2} + \frac{1}{2} m \dot{r}^2$$

$$u = \frac{1}{r} \qquad \dot{r} = \frac{dr}{d\phi}\frac{d\phi}{dt} = -\dot{\phi}\, r^2 \frac{du}{d\phi} = -\frac{J}{m}\frac{du}{d\phi}$$

$$du = -\frac{1}{r^2}\, dr$$

$$dr = -r^2 du$$

$$\left(\frac{du}{d\phi}\right)^2 + u^2 - \frac{2m}{J^2}(E + Au) = 0.$$

$$\left[(u-\frac{mA}{J^2})^2-\left(\frac{mA}{J^2}\right)^2 = (u-\frac{1}{r_0})^2-\frac{1}{r_0^2}\right]$$

$$r_0=\frac{J^2}{mA}$$

$$\left(\frac{du}{d\phi}\right)^2+(u-\frac{1}{r_0})^2-\frac{1}{r_0^2}-\frac{2mE}{J^2}=0.$$

$$\left(\frac{du}{d\phi}\right)^2=\frac{e^2}{r_0^2}-(u-\frac{1}{r_0})^2$$

$$\frac{e^2}{r_0^2}=\frac{2mE}{J^2}+\frac{1}{r_0^2}$$

$$\frac{du}{d\phi}=\left(\frac{e^2}{r_0^2}-(u-\frac{1}{r_0})^2\right)^{\frac{1}{2}}$$

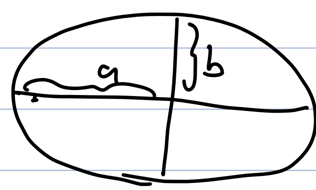
$$\int\frac{du}{\sqrt{\frac{e^2}{r_0^2}-(u-\frac{1}{r_0})^2}}=\int d\phi$$

$$\cos^{-1}\left(\frac{u-\frac{1}{r_0}}{\frac{e}{r_0}}\right)=\phi$$

1. $r\cos\phi=x$
2. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ← $\boxed{r_0=r(1+e\cos\phi)}$
3. →

$$a=\frac{r_0}{1-e^2}$$

$$b=\frac{r_0}{\ }$$



$\sqrt{1-e^2}$

$b = a^{\frac{1}{2}} r_0^{\frac{1}{2}}$

$$\text{period} = \frac{\pi ab \text{ (area)}}{\text{Rate of sweeping area.}} \Rightarrow \frac{\pi a a^{\frac{1}{2}} r_0^{\frac{1}{2}}}{\frac{1}{2} r^2 \dot{\phi}}$$

$$T \quad a \qquad = \frac{\pi r_0^{\frac{1}{2}} a^{\frac{3}{2}}}{\frac{J}{4m}} \sim a^{\frac{3}{2}}$$

$$\frac{1}{2} r^2 \dot{\phi} = \frac{J}{2m}$$

K2 K3