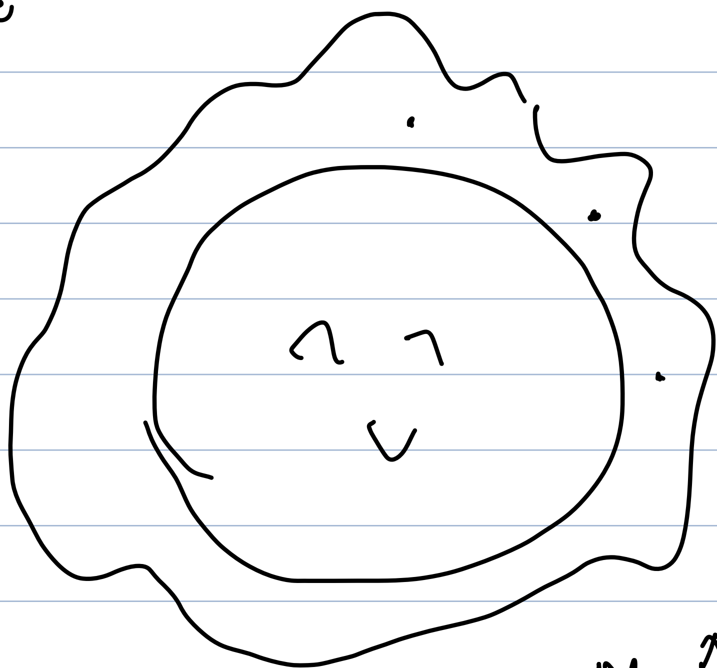
0 - 5 min 30 - 40 0 - 5 min

$$\underset{\text{number of particles}}{N}, \quad VP = NkT$$

volume, pressure, Boltzmann constant, temperature



oe

P

ionised-hydrogen

$$P = \frac{N}{V} kT$$

$$n$$

$$N = \frac{M_\odot}{m_H}$$

$$F = \frac{G m_1 m_2}{r^2} = \frac{G m^2}{r^2}$$

$$n = \frac{M_\odot}{\frac{4}{3}\pi r^3 m_H}$$

$$P = \frac{F}{4\pi r^2} = \frac{G m^2}{4\pi r^4}$$

$$\frac{G m^2}{4\pi r^4} = \boxed{\frac{M}{\frac{4}{3}\pi r^3 m_H}} kT$$

2×10^{33}

$$T = \frac{3 G m_\odot}{r m_H k}$$

gradient $\quad (\vec{\nabla f}) = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z} k$

divergence $\quad \vec{\nabla} \cdot \vec{F} = \left(\frac{\partial}{} \quad \frac{\partial}{} \quad \frac{\partial}{} \right)$

divergence $\nabla \cdot \vec{F} = \underbrace{\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \cdot (F_x, F_y, F_z)}$

$$= \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

curl $\vec{\nabla} \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$

laplacian

$$\Delta f = \nabla^2 f = (\nabla \cdot \nabla) f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\begin{cases} \nabla \cdot (\nabla \times A) = 0 \\ \nabla \times (\nabla \varphi) = 0 \\ \nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A \end{cases}$$

$\rightleftharpoons$ vector
scalar

Special relativity

(1) the laws of physics are invariant in all inertia frame of reference

$$\frac{d^2 x}{dt^2} = \frac{d^2 y}{dt^2} = \frac{d^2 z}{dt^2} = 0.$$

(2) speed of light, $c = 299792458$ m/s

in a vacuum is the same for all observer.

Newtonian mechanics

Maxwell → electromagnetism

↓

c

Newtonian gravity potential Φ

$$\vec{f} = -m_G \vec{\nabla} \Phi$$

→ passive gravitational mass

$$\vec{\nabla}^2 \Phi = 4\pi G \rho$$

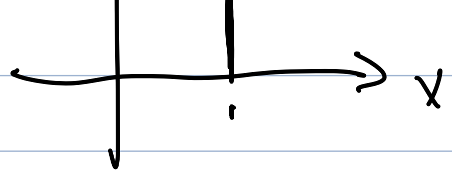
instantaneous change

⇒ travel faster than c

active gravitational mass.

$\delta(x-1)$

$$\rho(\vec{x}, t) = m_A \, \delta^{(3)}(\vec{x} - \vec{y}(t))$$



dirac delta function

position of the point particle.

$$\underline{\nabla^2 \Phi = 4\pi G m_A \, \delta^{(3)}(\vec{x}-\vec{y})}$$

$$\nabla \cdot \left(\frac{\vec{r}}{|r|^3}\right) \equiv 4\pi\, \delta(r)$$

$$\delta(\vec{r}) = \frac{1}{4\pi} \nabla \cdot \left(\frac{\vec{r}}{|r|^3}\right)$$

$$\delta^{(3)}(\vec{x}-\vec{y}) = \frac{1}{4\pi} \nabla \cdot \left(\frac{\vec{x}-\vec{y}}{|\vec{x}-\vec{y}|^3}\right)$$

$$\nabla^2 \Phi = \frac{\cancel{4\pi} G m_A}{\cancel{4\pi}} \nabla \cdot \left(\frac{\vec{x}-\vec{y}}{|\vec{x}-\vec{y}|^3}\right)$$

$$\nabla \Phi = G m_A \frac{(x-y)}{|x-y|^3}$$

$$f_{2\,on\,1} = - m_{G_1} G m_{A_2} \frac{(x-y)}{|x-y|^3}$$

$$f_{1\,on\,2} = - m_{G_2} G m_{A_1} \frac{(y-x)}{|y-x|^3}$$

$$\underline{m_{G_1} m_{A_2} = m_{G_2} m_{A_1}}$$

universality

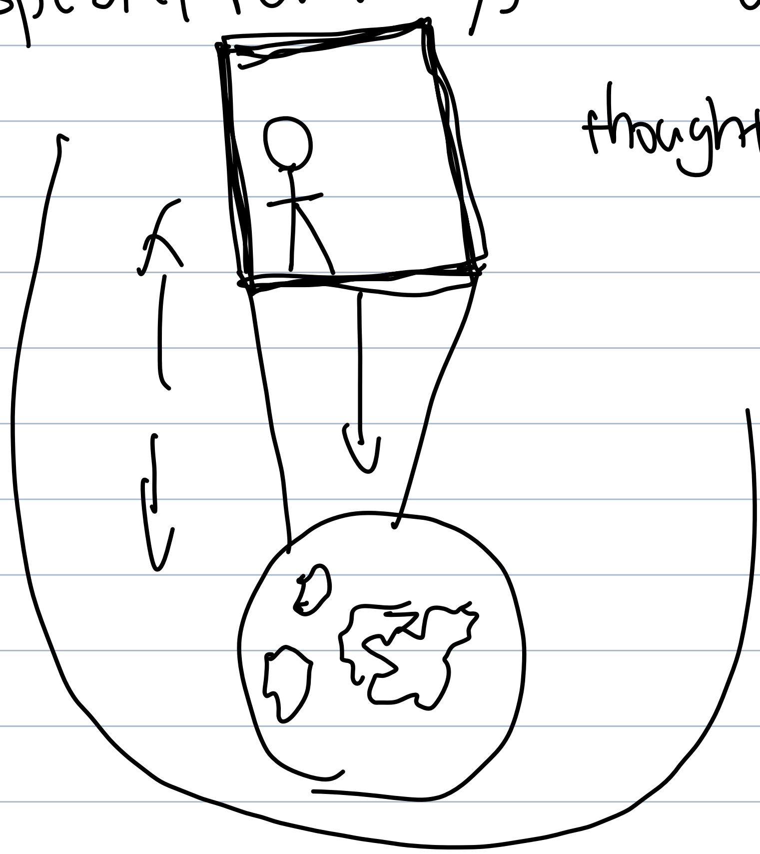
e m

weak equivalence principle

Freely - falling particles with negligible gravitational self - interaction follow the same path through space & time if they have the same initial position & velocity independent of their composition.

SR → GR (general relativity)
(special relativity)

acceleration ≈ gravity.
↓
thought experiment
<u>local</u>



Strong equivilency principle.

In an arbitrary gravitational field, all laws of physics in a free-falling non-rotating laboratory occupying a sufficiently small region of spacetime looks locally like special relativity

(with no gravity)